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Hybrid Inflation and the Moduli Problem

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Abstract

We revisit some questions in supersymmetric hybrid inflation (SHI). We analyze the amount of fine tuning required in various models, the problem of decay at the end of inflation and the generation of baryons after inflation. We find that the most natural setting for HI is in supersymmetric models with non-renormalizable couplings. Furthermore, we argue that almost inevitably, one of the fields involved is a modulus, with Planck scale variation. The resulting moduli problem can be solved in two ways: either by a massive modulus (which requires some fine tuning), or an enhanced symmetry point, in which the moduli becomes strongly coupled to the Standard Model. Various possibilities for baryon production are discussed.

1 Introduction

The hypothesis that the universe underwent a period of inflation early in its history has received striking experimental confirmation. From the perspective of fundamental physics, this is surely a clue to physics at extremely high energies. String theory moduli would seem to be natural inflaton candidates. If one considers a single field, and supposes that the curvature of the potential is everywhere of order TeV, the number of e-foldings is naturally of order one. Sufficient inflation can arise if one is willing to suppose a "tuning" not worse than about 1%. On the other hand, for a potential of this size, the resulting fluctuations are far too small, of order $\frac{m_{3/2}}{M_p}$.

At least two alternative proposals have been put forth to resolve this problem. One possibility is that some moduli are simply far more massive than a TeV; at their minimum, supersymmetry is restored and the cosmological constant vanishes[1]. The second possibility is that there is more than one field involved in inflation. This is known as "hybrid inflation"[2, 3]. Guth and Randall pointed out that hybrid inflation is particularly natural in the framework of supersymmetry[4, 5], where it provides a setting in which inflation can occur when $H \sim$ Tev. In a supersymmetric context, adequate inflation and a suitable fluctuation spectrum, as we will review, can be achieved with order 1% fine tuning[6]. Guth and Randall noted that such inflation works most naturally if some of the fields involved are flat directions of the theory. In this note, we will pursue the properties of these fields further. We will see that these fields are necessarily moduli with Planck scale variation, as in string theory. At generic points in the moduli space, these fields couple very weakly to ordinary matter. As a result, in its most appealing form, supersymmetric hybrid inflation (SHI) suffers from a moduli problem.

In this note, we explain how and why this problem arises. We then discuss two widely studied solutions to the moduli problem: massive moduli and enhanced symmetries. Both have a rather natural implementation in SHI. We will argue that in SHI, the starting point of the field evolution is probably a point of enhanced symmetry[1]. If the final point is not, then the resulting moduli problem might be cured if the modulus is quite massive, of order 100 TeV. This fine tuning has been widely discussed, and is troubling. In the context of SHI, however, such a tuning has a possible virtue: it facilitates achieving adequate inflation and suitable density fluctuations. So this is suggestive that one might be on the right track. Alternatively, the endpoint of the moduli evolution might also be a point of enhanced symmetry. This may seem

[1]In which case one might worry about topological defects such as domain walls and cosmic strings. But we will see that the structure of hybrid inflation models naturally avoids such difficulties.

a contrived possibility except that in string theory, as well as many supergravity Lagrangians, there exist moduli spaces with multiple points of enhanced symmetry. One of these, we will see, could explain the existence of the inflaton with its requisite couplings to the modulus; the other could resolve the moduli problem. Solving the moduli problem with an enhanced symmetry point does not entail any fine-tuning.

In both cases, the Affleck-Dine (AD) mechanism[7, 8, 9, 10] provides a natural way to understand the origin of baryon number, although other Baryogenesis scenarios can be applied.

In the next section, we review the hybrid inflation hypothesis and investigate how much fine tuning is necessary to comply with observations. We show that the amount of fine tuning is sensitive to couplings between the inflaton and the waterfall field and for certain non-renormalizable couplings a mild 1% tuning is sufficient. In section 3 we explain why moduli almost inevitably play an essential role, and review the cosmological moduli problem. In section 4 and 5, we discuss two possible solutions to the moduli problems of these models: enhanced symmetries and massive moduli. The four order of magnitude hierarchy (measured in $m^2/m_{3/2}^2$) needed in the latter to obtain a moduli with mass 100 TeV automatically implements the 1% fine tuning needed for inflation. Section 6 contains some remarks on the efficacy of late inflation in solving the moduli problem. We conclude in section seven.

2 Fine Tuning in Models of Hybrid Inflation

Hybrid inflation in supersymmetric models, and its virtues (and possible problems) are best illustrated by some examples. These are based on papers of Guth and Randall[4, 5], Lyth[11] and others. Below we begin by reviewing the idea of hybrid inflation. We then go on to argue that hybrid inflation suffers from a mild fine tuning problem which can be parametrized by a single dimensionless parameter $\mathcal{N}$. All other fine tuning issues are resolved if one considers non-renormalizable couplings between ϕ and χ.

2.1 A Renormalizable Model

Consider a (supersymmetric) theory with two fields, χ and ϕ. χ is a modulus, by which we mean simply that it has no potential in the absence of supersymmetry breaking. Near the origin, we first study a superpotential of the form:

$$W = \frac{1}{2}\lambda\chi\phi\phi \tag{1}$$

Note that this sort of structure is familiar in string theory. At points of enhanced symmetry, one typically has a variety of light fields which couple to moduli through the superpotential or gauge interactions. Far away from these points, fields which can couple to moduli will generally gain Planck scale masses, if not through couplings of the form of eqn. (1), then through couplings like $\chi^n\phi\phi$. Including soft breaking terms, the potential takes the form (in the region of small χ):

$$V = V_o - m_\chi^2|\chi|^2 + m_\phi^2|\phi|^2 + |\lambda\phi\chi|^2 + \ldots \tag{2}$$

V_o is assumed to be of order the scale typical of gravity-mediated models, $V_o = \mathcal{N} m_\chi^2 M^2$ where M is the reduced Planck mass $M = M_{\rm Pl}/\sqrt{8\pi}$ and $\mathcal{N}$ is our fine tuning parameter, to be evaluated below. This corresponds to the assumption that χ sits a distance of order M from its minimum. At $\chi = 0$, ϕ might also be a modulus, but this is not necessary for the mechanism to work. If the superpotential contains ϕ^3 terms, for example, this will not disrupt the mechanism described below, provided the coupling is not too large (such terms can be prohibited by symmetries).

The basic picture is that ϕ starts away from its minimum at the origin. As long as $\lambda\phi > m_\chi$, χ is locked near the origin. During this period, the energy is dominated by V_o, and inflation occurs. Once the curvature of the χ potential becomes negative, χ rolls towards its true minimum, and inflation ends.

A remark is now in order. Enhanced symmetry points may involve both continuous and discrete symmetries. Continuous gauge symmetries don't pose cosmological problems, so long as the U(1) of the Standard model is not embedded in a simple group at $\chi = 0$. If it is, one has to worry about possible production of monopoles after inflation. Discrete symmetries, on the other hand, would seem to raise the specter of domain walls. If χ is zero during inflation, its subsequent motion may break some of the discrete symmetries. However, this problem does not arise in this case or in the other models we will discuss. First, if the inflaton, ϕ, transforms under the discrete symmetry broken by χ, the discrete symmetry is already broken during inflation, and our observable universe lies within a single domain. Furthermore, if we consider our superpotential for χ and ϕ, we must include soft breaking effects. Thus we expect the potential to include a term $m_{3/2}W$. This term induces a VEV for χ (of order the VEV of ϕ), which means that χ also breaks the symmetry by a significant amount. One can check that the χ VEV does not spoil the slow roll of ϕ, or any of the other essential features of the dynamics we describe now. Thus there appears to be no problem of domain walls.

One of our goals is to explore fine-tuning in this model. We will see that the observational

constraints on the model are not met within the framework of renormalizable superpotential unless one allows a large amount of fine tuning (we are therefore led to non-renormalizable models which require much less fine tuning). The parameters that we use to quantify fine tuning are

$$\mathcal{N}_\chi = m_\chi^2/m_{3/2}^2, \quad \mathcal{N}_\phi = m_\phi^2/m_{3/2}^2, \quad \mathcal{N} = V_0/M^2 m_\chi^2, \quad \lambda \tag{3}$$

With the exception of section 5, we assume that $\mathcal{N}_{\chi,\phi}$ are of order 1, pushing the discussion of fine tuning to the remaining parameter.

The observational constraints determine these parameter as follows. Let us first compute the number of e-foldings. Assuming slow roll for ϕ, this is given by

$$\begin{aligned} N &= \int dt H(t) = -\int d\phi \frac{V_o}{M^2 V'(\phi)} \\ &= -\mathcal{N} \ln(\phi_f/\phi_i). \end{aligned} \tag{4}$$

So we require $\mathcal{N} \gtrsim 60$.

Next consider the spectral index given by

$$n_s = 1 - 3M^2 \left(\frac{V'}{V}\right)^2 + 2M^2 \frac{V''}{V}. \tag{5}$$

Since during inflation χ is heavy, one may integrate it out to be left with an effective potential for ϕ. As usual, the second term on the LHS is much smaller than the third. One therefore obtains,

$$n_s \simeq 1 + \frac{2}{\mathcal{N}} \tag{6}$$

which is larger than one. Thus the more e-foldings we have, the closer n_s is to unity. This point of view relates the smallness of the slow-roll parameters (and therefore the scale independence of the spectrum) to the requirement of sufficiently long inflation. For $\mathcal{N} > 60$, the above agrees well with observations.

Finally, we study the size of fluctuations. The standard relation is

$$\frac{\delta\rho}{\rho} = \frac{V^{3/2}}{M^3 V'} \simeq \mathcal{N}^{3/2} \frac{m_\chi}{\phi_{60}} \tag{7}$$

where ϕ_{60} denotes the value of ϕ 60 e-foldings before the end of inflation. Thus one needs $\phi_{60} \gtrsim 10^6 m_\chi$ which for the potential (1) means $\lambda \lesssim 10^{-6}$. We view this as a fine tuning which renders the renormalizable model unattractive.

2.2 A Non-renormalizable Model

The problem of fine tuning can be mitigated by considering a non-renormalizable interaction such as[2]

$$W = \lambda\chi\frac{\phi^n}{M^{n-2}}, \tag{8}$$

with the same soft SUSY breaking as in (2). Inflation occurs in the vicinity of the value of ϕ where the positive mass for χ arising from $|\frac{\partial W}{\partial\phi}|^2$ is slightly larger than the tachyonic mass of χ arising from SUSY breaking.

The number of e-folding remains $\mathcal{N}$ and the spectral index also remains as in equation (6). However, the constraint from the size of fluctuations changes. At 60 e-foldings before the end of inflation

$$\phi_{60} \simeq M\left(\frac{m_\chi}{n\lambda M}\right)^{1/(n-1)} \tag{9}$$

and the constraint of $\delta\rho/\rho \simeq 5\times 10^{-4}$ is translated to

$$\mathcal{N}^{3/2}\lambda^{1/(n-1)} \simeq 10^{-3.5+15(n-2)/(n-1)}. \tag{10}$$

Thus for $n \geq 3$ we can set $\lambda \sim 1$ and the constraint is translated to the number of e-foldings without further fine tuning. For $n = 3$ and λ of order one, we find $\mathcal{N} \simeq 5\times 10^2$ which is sufficient for the required 60 e-foldings

For larger n's one requires a larger fine-tuning: $n = 4 \rightarrow \mathcal{N} \sim 10^4$ etc. As n increases, the VEV of ϕ in which inflation occurs increases as well. As we will discuss briefly in section 3, this can be used to constrain n - if ϕ does not decay to Standard model fields before matter-radiation equality (which implies either large enough mass, or large enough couplings to the SM), then $n \geq 5$ is excluded.

In [4], it was necessary to impose a hierarchy between m_ϕ, $m_{3/2}$ and m_χ. Since m_ϕ and m_χ are both expected to be proportional to $m_{3/2}$ one can view this as fine tuning of $\mathcal{N}_{\chi,\phi}$. The need for this hierarchy originate from the evolution of χ. The authors show that unless inflation ends sufficiently rapidly, χ creates unacceptably large density fluctuations. The requirement of rapid ending requires $m_\phi \ll m_{3/2} \ll m_\chi$. As we now argue, this hierarchy is not necessary when one takes into account all of the expected soft breaking terms.

Indeed, given the superpotential (8), it is inevitable that soft breaking terms of the form $m_{3/2}W+$ c.c. will appear at low energy after supersymmetry breaking. Such tadpole terms

[2] If ϕ is a standard model field, then it should be thought of as some linear combination of fields of definite charge so that these expressions are gauge invariant.

trigger a VEV for χ of order ϕ (assuming $\lambda = \mathcal{O}(1)$) already during inflation. Fluctuations around this central value are of order[3] H and are therefore small $(H/\chi \simeq (m_{3/2}/M)^{\frac{n-2}{n-1}} \ll 1$ for $n \geq 3$). Thus one expects the fluctuations of χ to be of the same order as the fluctuations of ϕ both of which agree with observations. This differs considerably from the situation where χ has no VEV and fluctuations may be large (which is the case in [4]).

3 The Moduli Problem in Hybrid Inflation

While it is not always clearly stated, the assumptions of the previous section almost inevitably imply that χ is a modulus. By modulus, here, we mean a field with no potential in the limit of vanishing supersymmetry breaking. As a prototype, suppose there were terms in the superpotential of the form

$$W_\chi = \frac{\chi^{m+3}}{M^m}. \tag{11}$$

Suppose, again, that $\chi = 0$ is the region in which inflation occurs, and at this point χ has soft terms, $-m_{3/2}^2|\chi|^2 + m_{3/2}W$. So the minimum of the χ potential is at a scale less than M,

$$\chi \approx M \left(\frac{m_{3/2}}{M}\right)^{\frac{1}{m+1}}. \tag{12}$$

The energy shift between the symmetric minimum and this final minimum is then much less than $m_{3/2}^2 M^2$:

$$\Delta V \approx m_{3/2}^2 M^2 \left(\frac{m_{3/2}^2}{M^2}\right)^{\frac{1}{m+1}}. \tag{13}$$

It seems difficult to build models of inflation with modest values of m, at least with the usual assumptions about the scales of supersymmetry breaking. For example, plugging in eqn. (4), the number of e-foldings will be suppressed by

$$\epsilon = \left(\frac{m_{3/2}^2}{M^2}\right)^{\frac{1}{m+1}}. \tag{14}$$

In [4], flat directions of the MSSM are discussed extensively. But for all of these, the gauge symmetries alone are not enough to prevent corrections of the type of eqn. (11). Additional

[3] Here one assumes χ is moving fast enough to catch up with its minimum. This is true since $\dot\chi = (2-n)\dot\phi$ and the minimum depends on ϕ.

symmetries *can* render these directions exactly flat. Discrete R symmetries can forbid infinite numbers of operators and yield such flat directions[12]. Moreover, in string theory, it is well known that such flat directions can sometimes arise by what appears to be, from a low energy point of view, an accident. The absence of such terms might have implications for proton decay.

The dynamics of the system after inflation is the interplay of two points - $\chi = 0$ where inflation occurs, and the true minimum of the system, which is a distinct point to which χ rolls after inflation. One expects $\chi = 0$ to be a point of enhanced symmetry. The simplest reason is that enhanced discrete symmetries are needed to obtain a potential which is suitable for inflation. A related argument is that it is important that near the origin, χ couples to the light field ϕ in the superpotential. This is most natural if the point $\chi = 0$ is a point of enhanced symmetry, and the light ϕ is associated with this enhanced symmetry.

Given that χ is a modulus, and that around the true minimum its couplings to Standard Model fields are likely to be Planck suppressed, it potentially suffers from the standard moduli problem of string cosmology[13]. Once ϕ is small enough, the modulus, χ, begins to roll towards its minimum and inflation ends. Away from $\chi = 0$, the potential for χ is assumed to take the form:

$$V(\chi) = m_{3/2}^2 M^2 f(\chi/M) \tag{15}$$

χ reaches its minimum in roughly a Hubble time, and begins to oscillate. It behaves like pressureless dust, with $p = 0$, and dominates the energy density of the universe until it decays. It is usually assumed that the width of such a particle is of order

$$\Gamma = \frac{m_\chi^3}{M^2} = 10^{-27} \text{ GeV}. \tag{16}$$

This corresponds to a reheat temperature of order 10 KeV. This spoils the successes of the standard cosmology.

Reference [4] considered the possibility that χ is one of the approximate flat directions of the MSSM. The authors did not address the question of the required flatness of the χ potential. As we indicated, the potential must be extremely flat; this might be accounted for by symmetries or by some stringy phenomenon. We have suggested that the lightness of ϕ during inflation might result from an enhanced symmetry at this point in the χ potential. As we will discuss later, in string theory it sometimes occurs that a moduli space has multiple enhanced symmetry points. If this accounts for the features of χ, then the χ couplings to the SM need not be suppressed at the minimum, and, as we will shortly explain, there would be no moduli problem.

It should be noted that ϕ might have a moduli problem of its own. As n increases in equation (8), the VEV of ϕ in which inflation occurs is pushed to higher values, and more energy is stored in ϕ. If $n > 4$ and ϕ is coupled to the SM only with M-suppressed couplings, it will have its own moduli problem, irrespectively of how the χ moduli problem is resolved. The solutions to the ϕ moduli problem are either the ones that we will discuss for χ (with additional fine tuning if ϕ is a massive moduli), or one concludes that $n < 5$.

A final remark is in order. In the following two sections we present two possible solutions to the cosmological moduli problem. Both solutions involve a long epoch in which the universe is matter dominated before χ finally decays. During this time, density fluctuations grow and one may worry of an excess density of primordial black holes and/or Q-ball formation. Moreover, in the enhanced symmetry scenario, a positive or negative pressure might build up due to running of the mass parameter[9]. The former could prevent the formation of these objects while the latter would allow Q-ball generation; these Q-balls could act as dark matter and create the baryon asymmetry[14]. In either case, this would constrain the low energy theory considerably. We will return to this point in future work[15].

4 Enhanced Symmetries

Since χ couples to massless fields, it is natural to suppose that $\chi = 0$ is a point of enhanced symmetry (although there could be points in the moduli space where, accidentally, some fields are massless). Such an enhanced symmetry is necessary for the structure hybrid inflation potential. This also makes it natural that the χ field starts near the origin. Suppose, however, that the true minimum of the χ potential lies at another point of enhanced symmetry. Then there is no problem with this modulus; its lifetime is of order $m_{3/2}$ (modulo factors of coupling constant). In addition, χ might be some linear combination of fields of the MSSM, i.e. a flat direction of the MSSM. So motion in this direction could lead to production of baryons. Indeed, one might worry that unless CP violation is very small, there will be too many baryons. We will see that this is the case.

It is easy to see how discrete R symmetries can make some of the flat directions of the MSSM exact. Suppose, for example, that one has a Z_N symmetry ($N > 3$), under which the Higgs transform with phase $\alpha = e^{2\pi i/N}$, while the quarks and leptons are neutral. Suppose that the superpotential, W, transforms with phase α. In addition, suppose that there is a Z_2 symmetry (non-R) under which the quarks and leptons are odd, and all other fields are

even. Then the usual Yukawa couplings are all allowed. (The μ term vanishes as well; when supersymmetry is broken, the R symmetry is broken as well, and a μ and B_μ terms may be generated). But terms in the superpotential involving powers of the various quark and lepton invariants, $\bar{u}\bar{d}\bar{d}$, $QQQL$, $\bar{u}\bar{u}\bar{d}\bar{e}$, etc., are all forbidden. Invariants involving Higgs fields are permitted, but typically must involve several fields. It is easy to see that there are numerous exact flat directions: $QQQL$, $\bar{u}\bar{d}\bar{d}$, etc.

One might object that these symmetries have anomalies. Since in an underlying theory, one expects any discrete symmetries are gauge symmetries, this might seem inconsistent. But it is not difficult to anomaly free variants with many flat directions. For example, if the third generation leptons, L_3 and $\bar{e}_3$ transform with phase α^{-2} and α^2, respectively, there are no anomalies, yet one can check that there are many flat directions. Interestingly, in this case, one can check as well that all baryon violating dimension four and five operators are forbidden. To account for neutrino mass, it may be necessary to modify these assignments, but this example illustrates that anomalies are not an obstacle.

One might also ask: how plausible is it to find more than one point of enhanced symmetry in a moduli space. Certainly, in toroidal compactifications of weakly coupled string theories, there are many points of enhanced symmetry at which *all moduli but the dilaton* are charged. E.g. for the heterotic string on T^6, there are points of $SU(2)^6$ and $SU(3)^3$ symmetry. In addition, at these points, one has unbroken discrete symmetries. It is easy to check that all moduli but the dilaton transform under one or more of these symmetries.

4.1 Reheating and Barygenesis

4.1.1 Moduli Space of Real Co-Dimension> 1

In the enhanced symmetry scenario the field χ becomes part of the SM near its true minima. We will see that the cosmological evolution of the model does not depend sensitively on where χ sits in the SM. The features of this field that we use are

1. In general, it could be that the inflationary point in field space is characterized by several moduli that are fixed during inflation. We will denote these by $\rho_i,\ i = 1..N-1$. As we move to large values of χ, it mixes with ρ_i and they become indistinguishable. We will denote the entire collection by $\rho_i,\ i = 1..N$. The true minima is assumed to be an isolated point in this N dimensional space.

2. The fields ρ_i becomes strongly coupled to the SM only within a circle of size $m_{\rho_i} \sim m_{3/2}$ in this moduli space. At large ρ, the fields to which these moduli couple have mass of order ρ; interactions of the ρ fields with remaining light fields are suppressed by factors[4] of ρ^{-1}.

3. Once the amplitude of the field ρ_i becomes small enough, these fields decay. Equating decay rates and expansion rates, this typically occurs when the amplitude is of order $10^6 - 10^7$ GeV. The reheat temperature is of order $10^2 m_\rho \sim 10^5$ GeV.

For now we will keep the distinction between ϕ and the ρ_i.

When working in the supersymmetric framework, the fields ρ_i parameterize a complex manifold and the real co-dimensionality of true minima is necessarily greater than one (still, the co-dimension 1 case is interesting, and we will discuss it in the next subsection). For this larger co-dimension case, it is straightforward to see that the reheat temperature is typically two or three orders of magnitude above $m_{3/2}$. The decay rate of the ρ_i fields is $\Gamma \approx \frac{m^3\alpha^2}{\rho^2}$. This leads to a reheat temperature of order

$$T_{rh} \approx m(\alpha^2 M_p/m)^{1/6}. \tag{17}$$

and quite a large baryon number

$$n_B/s \sim 10^{-2} \times \text{ CP} - \text{phases}. \tag{18}$$

To understand this estimate, suppose that the ρ_i fields have baryon number q_i. Parameterizing, $\rho_i = |\rho_i| e^{i\theta_i}$, the baryon number is then given by

$$n_B = \Sigma_i q_i \int d^3x \, (\rho_i^* \dot{\rho}_i - \dot{\rho}_i^* \rho_i) \simeq \Sigma_i q_i \int d^3x |\rho_i|^2 \dot{\theta}_i \tag{19}$$

which leads to n_B/s of the order,

$$\frac{n_B}{s} \simeq \left(\Sigma_i q_i |\rho_i|^2 \dot{\theta}_i \right) \frac{T_R}{E_\rho} \tag{20}$$

where E_ρ is the total energy in ρ_i. At the time of decay, the above becomes $n_B/s \simeq \left(\Sigma_i q_i \dot{\theta}_i \right) / m_{3/2} (10^{-2})$. Here $\dot{\theta}_i$ depend on the baryon violating interactions. For example the Kahler potential is likely to break the $U(1)_B$ through terms such as

$$K = \rho_i \bar{\rho}_i \left(1 + \frac{\rho_i^n}{M^n} + \frac{\bar{\rho}_i^n}{M^n} + \ldots \right) + \ldots \tag{21}$$

[4] If $\delta\rho$ is the canonically normalized fluctuation of the field, and ρ_0 is the VEV, then the interaction to the SM is through $(\delta\rho/\rho_0)$, i.e., $\mathcal{L} = \ldots + log(\rho) * \mathcal{O}_{(SM)}$

Immediately after inflation, when $\rho_i \sim M$, the potential is such that the ρ_i phase gets a "kick", generating some $\dot{\theta}_i \sim m_{3/2}$. Thus unless one tunes the CP and baryon violating terms to get $\dot{\theta}_i/m_{3/2} \ll 1$ at the end of inflation, one ends up with order one baryon number. Adequately suppressing the baryon number seems to require tuning, so this does not seem a promising mechanism.

One may hope that the inflaton field itself may serve as the AD field. However, checking the details of the model shows that this is not the case. ϕ will produce the largest amount of baryon number under two conditions: 1) it decays to SM only when ρ_i decays and not earlier and 2) there are terms in the superpotential which produce a non-zero $\dot{\theta}$. However, even if both conditions are fulfilled, since inflation ends when ϕ/M is small (eq. (9)), one obtains $E_\phi/E_\chi \ll 1$ and an extremely eccentric orbit with $\dot{\theta}/m_{3/2} \ll 1$ both of which suppresses the baryon number below the observed value.

If we are not willing to accept fine tuning, we can enumerate other possibilities in the case that the moduli are not weakly coupled in their ground state, and the reheat temperature is $m_{3/2}$ or larger:

1. An additional SM field flat direction which carries baryon number and is not a modulus has a VEV. Using discrete symmetries one can then arrange the appropriate Baryon number via an AD mechanism[8, 10].
2. Other possibilities might include EW baryogenesis. Although the usual SM-EW baryogenesis fails to generate a sufficient amount of baryons, many extensions to the SM work fine. The simplest, is the MSSM-EW baryogenesis which has more CP phases and which enhances the strength of the phase transition[16, 17, 18, 19]. Other examples of extensions of the SM or the MSSM exist (see for example [20, 21]).
3. Leptogenesis at the TeV scale can be ignited[22].

4.1.2 Moduli Space of Co-dimension=1

Although less interesting in the supersymmetric context[5], the case of co-dimension one is special since the reheat temperature is significantly higher. The physical reason for this is that a one dimensional field must go through the regime of strong coupling in each oscillation and thereby

[5]The co-dimension 1 dynamics is relevant for the supersymmetric case if the motion of the supersymmetric complex field around its minima is highly eccentric. This can be achieved but requires fine tuning.

the release of energy begins much sooner. Since the field is real, it does not carry baryon number.

In this section we will use a yet simpler model for the coupling of χ to the SM. χ is now a real field. The assumption is that there is a real modulus χ with mass $m_{3/2}$ that starts at a VEV of order M. Around VEV zero of the field there is small region of size $m_{3/2}$ in field space in which the field is strongly interacting with the SM, and the decay rate there is $\Gamma \sim m_{3/2}$ in the sense that in flat space the loss of energy is

$$\partial_t E_\chi = -\Gamma E_\chi. \tag{22}$$

We wish to compute the reheat temperature. The EOM for the field reads,

$$\ddot{\chi} + 3H\dot{\chi} + \theta(m_{3/2}^2 - \chi^2)\Gamma\dot{\chi}_i + m_{3/2}^2\chi_i = 0. \tag{23}$$

Multiplying the above by $\dot{\chi}$ and averaging over one cycle we find

$$\partial_t E_\chi + 3HE_\chi + m_{3/2}^2\Gamma\sqrt{E_\chi} = 0, \tag{24}$$

$$\partial_t E_{\mathrm{SM}} + 4HE_{\mathrm{SM}} - m_{3/2}^2\Gamma\sqrt{E_\chi} = 0, \tag{25}$$

where we've defined E_χ (E_{SM}) to be the energy in χ (SM).

Assuming that the energy is dominated by χ we can solve the 1st equation. Changing variables to $a = e^\eta$, the equation becomes

$$\partial_\eta(e^{3\eta}E_\chi) = -\sqrt{3}e^{3\eta}m_{3/2}^2\Gamma M \tag{26}$$

and the solution is

$$E_\chi = e^{-3\eta}m_{3/2}^2M^2 - \left(1 - e^{-3\eta}\right)\frac{1}{\sqrt{3}}m_{3/2}^2\Gamma M. \tag{27}$$

The shift to a negative asymptotic number is the advantage relative to the standard moduli issue. This energy goes to zero when the scale factor is increased by roughly

$$a/a_0 \simeq (M/\Gamma)^{1/3}. \tag{28}$$

Equation (27) is to be trusted only until $E_\chi = E_{SM}$, but this occurs slightly before (28).

Plugging this into the equation for the energy in the SM we get the equation (again in the epoch in which E_χ dominates)

$$\partial_\eta E_{\mathrm{SM}} + 4E_{\mathrm{SM}} - \sqrt{3}m_{3/2}^2\Gamma M = 0, \tag{29}$$

and the solution (with $E_{SM} = 0$ as initial condition) is

$$E_{\rm SM} = \frac{\sqrt{3}}{4}\left(1 - e^{-4\eta}\right) m_{3/2}^2 \Gamma M. \tag{30}$$

One can evaluate when the energy in the SM and in χ are approximately the same. This turns out to be very close to the point in which the amplitude of χ is $m_{3/2}$. Thus when they are equal the field χ is already strongly interacting and there is no need to discuss the SM dominated universe with a distinguished χ component. Finally, the reheat temperature is approximately $\left(m_{3/2}^2 \Gamma M\right)^{1/4}$ or for $\Gamma \sim m_{3/2}$, $T_{\rm Re} \simeq \left(m_{3/2}^3 M\right)^{1/4} \simeq 10^7$ GeV. Such reheat temperature cannot ignite the usual leptogenesis which requires $T_{\rm Re} > 10^8 - 10^{10}$ GeV [23, 24, 25]. Other possibilities are (non-supersymmetric) low energy variants of EW-baryogenesis[20] and possibly low-scale leptogenesis[22, 26, 27] (although these models mostly work in the supersymmetric framework and are therefore less relevant).

5 Massive moduli

As has often been noted[13], a modulus with mass of order 100 TeV will, when it decays, reheat the universe to temperatures of order 10 MeV, restarting nucleosynthesis. This large mass is usually felt to represent a disturbing fine tuning. But in the context of slow roll inflation, as presently understood, some fine tuning appears inevitable. Although as we now argue the fine tuning required to solve the moduli problem is slightly more significant, it is still interesting that this fine tuning of the mass can simultaneously solve the moduli problem and yield a suitable number of e-foldings and a satisfactory fluctuation spectrum, without further fine tunings.

To be more precise, let us restore our three fine tuning parameters $\mathcal{N}_\phi$, $\mathcal{N}_\chi$ and $\mathcal{N}$. The moduli problem requires $\mathcal{N}_\chi \simeq 10^4$ and we require no further fine tuning in V_0, namely, $\mathcal{N} = 1$. With this, the observables from inflation take the form,

$$N = \frac{\mathcal{N}_\chi}{\mathcal{N}_\phi} \tag{31}$$

$$n_s = 1 + \frac{2\mathcal{N}_\phi}{\mathcal{N}_\chi}, \tag{32}$$

and

$$\frac{\delta\rho}{\rho} = \frac{\mathcal{N}_\chi}{\mathcal{N}_\phi}\left(\frac{m_\chi}{M}\right)^{n-2/n-1} \tag{33}$$

So for $n = 3$ one requires m_ϕ to have a mass only slightly above $m_{3/2}$, namely $\mathcal{N}_\phi \simeq 10$, in order to explain the fluctuation spectrum. For $n = 4$, m_ϕ must be anomalously small with $\mathcal{N}_\phi \simeq 10^{-1}$.

In the heavy modulus scenario, baryons can be produced through modulus decays, but this requires significant violation of R-parity, which may be problematic. An alternative possibility is AD baryogenesis, which is quite natural in such a framework. This requires that, in addition to the heavy moduli field(s) χ, that some of the SM flat directions be *very* (possibly exactly) flat. Call this (these) directions Φ. For simplicity, suppose that it is exactly flat. It is possible – and natural – that, when $\chi = 0$, the minimum of the Φ potential is far from the origin. When when χ begins to roll towards its minimum, so does Φ. A baryon asymmetry will be produced in the motion of Φ. If the Φ potential is flat, and the initial Φ amplitude is of order M_p, this will be of order one per Φ particle, times CP violating phases. Note that the number of Φ particles is comparable to the number of χ particles.

Initially, there will be a period where Φ and χ are oscillating simultaneously. But near the origin, Φ couples to light fields, so it will decay earlier. The baryon number can be estimated by noting that, just as there is of order one baryon per Φ particle, there is of order one baryon per χ. Since the reheating temperature is of order 10^{-2} GeV, the number of photons per χ particle is $10^5/10^{-2} = 10^7$. So the baryon per photon ratio is of order 10^{-7} times CP violating phases. So this is quite naturally in a reasonable ballpark. If Φ is an approximate modulus, it can still generate a suitable baryon number, provided that its potential is flat enough. The Φ decays can also lead to production of a suitable dark matter density[28]. A scenario which realizes these possibilities has been discussed in [29].

6 Weak Scale Inflation?

Weak scale inflation is often mentioned as a solution of the moduli problem, and one might try to invoke it here in order to understand solve the moduli problem in this context. Indeed, we could consider a theory with an extra field, Φ. If we tune the potential of the Φ field suitably, we could obtain additional inflation, now with very small fluctuations. As described by Randall and Thomas[30], this number should not be larger than about 25, so as not to spoil the successes of the earlier stage of inflation.

In our scenario, however, additional weak scale inflation (which is in fact the scale of hybrid inflation) is not a possible solution since solving the moduli problem for χ, introduces a new moduli problem for the weak scale inflaton. For that reason, weak scale inflation at its best, still leaves a modulus.

In fact, it is not widely appreciated that *weak scale inflation quite generally does not*

generally solve the moduli problem. The difficulty is that, even when $H \sim M_p$, the minimum of the moduli potential has no reason, in general, to coincide with its $H = 0$ minimum. Corrections to the moduli potential will include terms such as

$$H^2|\chi|^2 \tag{34}$$

One might hope that, during inflation, the modulus will be driven to the instantaneous minimum of its potential, and then will remain in the instantaneous minimum. But it is easy to see that this is not the case. Call $V(\chi; t)$ the time-dependent potential for χ, and χ_o the instantaneous minimum. Then writing $\chi = \chi_o + \delta\chi$, the equation of motion:

$$\ddot{\chi} + 3H\dot{\chi} + V'(\chi, t) = 0 \tag{35}$$

becomes

$$\ddot{\delta}\chi + 3H\dot{\delta}\chi + V''(\chi_o; t) = -(\ddot{\chi}_o + 3H\dot{\chi}_o). \tag{36}$$

The problem is that, when H is comparable to $m_{3/2}$, all of the derivatives are of the same order, and the source term leads to a large $\delta\chi$. Note that we need a *huge* suppression, by many orders of magnitude, of $\delta\chi$.

7 Conclusions

In this paper, we have revisited various issues in hybrid inflation. In particular, we examined how finely tuned the model must be; does it exhibit a moduli problem and how can this problem be solved; and what is the natural way to produce baryons after inflation.

We have argued that the amount of fine tuning is highly sensitive to the couplings between the inflaton ϕ and the waterfall field χ. We found that for a certain non-renormalizable couplings $W = \chi\phi^3/M$ there exist a single mildly tuned parameter $\mathcal{N} \gtrsim 5 \times 10^2$ which then allows for a sufficiently long inflation together with a correct spectral index and density perturbations. Taking into account also soft breaking terms, we showed that no further tuning is necessary to prevent χ from creating large fluctuations at the end of inflation. From this point of view, one relates the large number of e-foldings to the scale invariance of the spectrum.

Next we argued that χ is necessarily a modulus and therefore it decays late, creating a moduli problem after inflation. Two solutions were offered. The first involves enhanced symmetry points where one assumes that the moduli space consists of an inflationary point

and a SM point both of which have enhanced symmetries. This in particular means that χ is strongly coupled near the SM point and therefore has no moduli problem. Since it is most natural to work in the supersymmetric framework, the reheat temperature turns out to be quite low, $T_{\rm Re} \sim 10^5$ GeV. Furthermore, soft breaking terms ensure that domain walls are created after SUSY breaking and before inflation and therefore impose no cosmological problem.

The second possible solution is heavy moduli. Usually, this possibility seems disturbingly fine-tuned, but in the case of hybrid inflation this same tuning can explain the number of e-foldings and the size of the fluctuation spectrum. It is tempting to imagine that the tuning might have an anthropic explanation. This might fit, for example, into a scenario in which the landscape predicts supersymmetry, put forward in [29].

With such low reheat temperature, it is more difficult to create the baryon asymmetry. We argued that the most natural way is through the AD mechanism. However, in both cases we showed that χ cannot be the AD field and one must introduce another flat direction which carries baryon number.

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